EFFECT OF POROUS MEDIA ON HYDRAULIC JUMP CHARACTERISTICS BY USING SMOOTH PARTICLE HYDRODYNAMICS METHOD

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Abstract

In order to understand the effect of porous media on hydraulic jumps, a smoothed particle hydrodynamics (SPH) model is applied to investigate the characteristics of hydraulic jumps interacting with porous media. Various of porosities, including cases without an obstacle or with a solid obstacle or porous media are considered. The opening of a gate was altered to adjust the hydraulic jump. The conjugate depth ratio, bottom shear stress distribution, and energy dissipation are reported. In the present study, validations are in a good agreement with previous studies. Overall, the result showed that the average error between numerical and experimental data was less than 7.2 %. Energy dissipation is compared among cases with three porosities, with and without a solid obstacle. The porosity of 0.68 is found to dissipate more energy than do other porosities. Thus, porous media can be used to enhance energy dissipation of hydraulic jumps in an open channel. In conclusion, the proposed SPH model can simulate the effect of porous media on hydraulic jump characteristics.

Keywords: Ecological engineering; Free surface flow; Energy dissipation; Porosity; Computational fluid dynamics

1 Introduction

Several experimental studies have investigated the hydraulic performance of stilling basins [1]-[3]. Furthermore, many researchers used different methods to reduce energy dissipation in a hydraulic jump. Alikhani *et al.* [4] performed experimental studies to investigate the effects of a vertical end sill on dissipation of energy. Comparison of forced jump results of this study with free jump relationships confirmed up to 30 % reduction in the length of stilling basin where the sill was used to control a jump. A preliminary study on submerged jumps with baffle walls and blocks downstream of a sluice gate was conducted by Habibzadeh *et al.* [5], who observed that the energy dissipation efficiency of free jump with blocks is larger than that of free jumps. Barani *et al.* [6] investigated the energy dissipation of flow over stepped spillways of different step shapes and reported that the energy dissipation of flow on end sill and inclined stepped spillways are more than the plain one. Ead and Rajaratnam [7] studied the effect of the round corrugated bed on a hydraulic jump and indicated that the sequent depth decreases and the hydraulic jump length decreases 20% and 50%, respectively and that the bed shear stress on the corrugated bed was about 10 times that on a smooth bed. In a theoretical and experimental study of steady two-layer flow over a fixed two-dimensional obstacle by Lawrence [8], a significant amount of energy was dissipated (primarily in the lower layer) in an internal hydraulic jump. Kim *et al.* [9] studied hydraulic jump and energy dissipation due to a sluice gate by examining the hydraulic jump phenomena produced by a sluice gate; the authors proposed an effective method to dissipate energy in fluid flow and design criteria of device dissipating energy for the protection of a river bed with a movable weir. Their results indicated that the height significantly influenced energy dissipation. The most effective height can increase 10% of the downstream surface water depth - relevant to energy dissipation in free surface flow.

Ljubicic *et al.* [10] studied hydraulic jumps in adverse-slope stilling basins for stepped spillways. By using the momentum conservation law and experimental data, the authors derived a method for estimating characteristics such as hydraulic jumps. The authors also discussed the possibility of using a configuration of stepped spillways when a depth of the tail water was ineffective for hydraulic jumps stabilization. Their proposed method is in agreement with experimental data on closely comparison with the existing methods and thus could be used for the initial design of the stepped spillways.

Takatsu and Masuoka [11] proposed a bank of cylinders in a narrow gap as a model for the flow through porous media and performed the PIV and LIF to examine the microscopic flow field in porous media. The researchers found that the large vorticity at the throat produced such a vortex as the swirl flow, leading to the production intrinsic to the turbulent flow through porous media. Higuera *et al.* [12] conducted a numerical study of wave breaking on a porous bed and noted large shear stresses at the porous media interface and increased energy dissipation. Smeulders and van Dongen [13] studied the influence of a small amount of gas within the saturating liquid of a porous medium or acoustic wave propagation. Darcy dumping appeared to be the dominant mechanism, but the compressibility effect became equally important at high frequencies. Wang *et al.* [14] developed a numerical approach to model the flow in porous media by using the homogenization theory and reported that the multi-dimensional effect becomes crucial for sand when void ratios became larger and larger cross-channels thus produce greater resistance.

A straight-forward method to determine the characteristics of a hydraulic jump is the smoothed particle hydrodynamics (SPH) method. SPH provides several advantages compared with the usual limitations of Eulerian mesh-based methods. Liu and Liu [15] mentioned that SPH conserves mass exactly and has the strong capability to deal with a free surface and moving interface problems. For instance, it has been used for modeling dam break flow. Standard SPH was formulated for solving Navier-Stokes equations [16]. Recently, the progress in the applications of standard SPH to dam-break behavior modeling has been significant. Most of these are targeted on 2D coastal flow simulations, fluid-structure interaction (Antoci *et al.* [17]), wave generation (Yim *et al.* [18]), and wave breaking (Shao [19]). For practical engineering applications (i.e., hydraulic jump), only few studies have attempted to investigate this topic. Lopez *et al.* [20] found that SPH provides good average bottom pressure values in the jump influenced area, but a large dispersion was observed for instantaneous water height. It was considerably improved by introducing another turbulence model. An extensive validation compared with experimental results was investigated by Chang *et al.* [21] where the authors proposed a meshless numerical model to solve the shallow-water equation (SWE) for dam break flow in 1D and 2D open channels based on SPH. The authors used three benchmark problems including dam break flows through a rough flat channel, a rough bumpy channel with various downstream boundary conditions, and a non-prismatic channel as 1D problems. A realistic scale model of the Toce river in Italy was used for 2D problems. The SPH numerical results indicated that accurate performance was reached in the presence of shock discontinuity, shock front motion and hydraulic jumps. Federico *et al.* [22] proposed a particular initial configuration, adopted to keep the jump close to its initial position and avoid the use of any weir downstream. Various jumps were simulated at seven Froude numbers. Chern and Syamsuri [23] investigated the effect of a corrugated bed on hydraulic jumps by using the SPH method by considering various corrugated beds which are smooth, triangular, trapezoidal, and sinusoidal corrugated beds. The authors found that the sinusoidal bed can dissipate more energy than other beds. Jonsson *et al.* [24] proposed SPH modeling of hydraulic jumps, bulk parameters and free surface fluctuation by focusing on the general characteristics of hydraulic jumps when using messless SPH method. The specially focused on how spatial resolution of SPH particles affected the overall characteristics of hydraulic jump and conjugate depth and reported that the average standard deviation of free surface fluctuations was similar to that obtained for experimental data.

Although recent investigations have investigated hydraulic jumps, information on the effect of porous media on hydraulic jump characteristics has not been discussed. In this study, porous media, which can be built by local rocks in a cage, are proposed to reduce energy dissipation in a hydraulic jump problem. Unlike rigid concrete, they do not require cement from other places. Recycled materials such as plaster or construction demolition waste can also be used to build porous media [25]-[27]. This research can facilitate ecological engineering, which aims to preserve the environment and use local natural resources. Here, porous media are simulated using a cylinder array, and the number of cylinder is adjusted to fit the porosity value.

2 SPH Formulae for Incompressible Fluid Flow

2.1 *Fundamentals of SPH method*

A function is approximated by the summation interpolant in SPH. For example, Φ is a smooth function and its value at a particle a can be approximated by

$$\Phi(\mathbf{r}) = \sum_b m_b \frac{\phi_b}{\rho_b} w_{ab}, \tag{1}$$

where neighboring particles are denoted by b; m is particle mass and ρ is particle density. The weighting function or kernel, w, plays a vital role in SPH. It is determined by the distance $\mathbf{r}$ between particle a and neighboring particle b and also the smoothing length h. Details of SPH including choice of the weight function can be found in [16].

2.2 *Governing equations*

Since SPH is a particle method, the Lagrangian method is utilized to describe governing equations of fluid motion. Conservation laws of mass and momentum can then be expressed by

$$\frac{1}{\rho}\frac{\mathrm{D}\rho}{\mathrm{D}t} + \nabla \cdot \mathbf{u} = 0, \tag{2}$$

and

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\rho}\nabla P + \mathbf{g} + \nu_o \nabla^2 \mathbf{u} \tag{3}$$

where $\mathbf{u}$ is velocity vector, P is pressure, $\mathbf{g}$ is gravitational acceleration and ν_o is kinematic viscosity. Those two equations can be approximated by SPH. For a specified particle a, they become

$$\frac{\mathrm{D}\rho_a}{\mathrm{D}t} = -\sum_b m_b \mathbf{u}_{ab} \nabla_a w_{ab}. \tag{4}$$

and

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi \right) \nabla_a w_{ab} + \mathbf{g} \tag{5}$$

where $\nabla_a w_{ab}$ is the gradient of the kernel with respect to the position of particle a and P_k and ρ_k are pressure and density of particle k (evaluated at a or b), respectively.

$$\Pi_{ab} = \begin{pmatrix} \frac{-\alpha \bar{c}_{ab} \mu_{ab}}{\bar{\rho}_{ab}} & \mathbf{u}_{ab}\mathbf{r}_{ab} < 0 \\ 0 & \mathbf{u}_{ab}\mathbf{r}_{ab} > 0 \end{pmatrix}, \tag{6}$$

where α is an empirical coefficient, $\bar{c}_{ab} = (c_a + c_b)/2$, $\bar{\rho}_{ab} = (\rho_a + \rho_b)/2$ and $\mu_{ab} = \frac{h\mathbf{v}_{ab}\mathbf{r}_{ab}}{r_{ab}^2 + 0.01h^2}$, with $\mathbf{u}_{ab} = \mathbf{u}_a - \mathbf{u}_b$ and $\mathbf{r}_{ab} = \mathbf{r}_a - \mathbf{r}_b$, where $\mathbf{r}_k$ and $\mathbf{u}_k$ are the position and velocity corresponding to the particle k (a or b), respectively. The parameter α must be properly selected for a stable and accurate solution. In general, this parameter is $\mathcal{O}$ (10^{-2}) for free surface flows (*e.g.* Colagrossi [28]). Large values of α would delay the wave breaking phenomena as mentioned by Delorme *et al.* [29].

The repulsive force term is imposed in the boundaries to prevent particle penetration. The momentum equation is modified and become (see [30])

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2} + \Pi \right) \nabla_a w_{ab} + \sum_k f_{ak} + \mathbf{g}. \tag{7}$$

The force per unit mass, f_{ak}, is used between them. f_{ak} is defined in Eq. 7.

The weighting function affects the accuracy of the SPH method [31]. In this study, the quintic kernel function from multifarious possible kernel was used,

$$w_{ab} = w(\mathbf{r}_a - \mathbf{r}_b, h) = \alpha_N (1 - \frac{q}{2})^4 (2q + 1), 0 \le q \le 2, \tag{8}$$

where $q = r_{ij}/h$ and $\alpha_N = \frac{7}{4\pi h^2}$ for 2D, $\alpha_N = \frac{21}{16\pi h^3}$ for 3D. The tensile correction is automatically activated when using kernels with first derivatives that go to zero with decreasing interparticle spacing.

Total derivatives in Eqs (4) and (5) are needed to be solved by a time marching scheme. The Beeman time stepping algorithm is used. Capone *et al.* [31] indicated that the stability and accuracy of this algorithm in SPH is higher than that of the others. Chern and Syamsuri [23] compared the Beeman and Predictor-Corrector methods. Their results are in a good agreement with the analytical solution. It turns out that using the modified Beeman algorithm combined with the quintic kernel provides the best result.

After Eqs. (4) and (5) are solved, particle velocities are determined. Subsequently, particles are moved by the following equation:

$$\frac{\mathrm{d}\mathbf{r}_a}{\mathrm{d}t} = \mathbf{u}_a + \epsilon \sum_b m_b \left(\frac{\mathbf{u}_a - \mathbf{u}_b}{\rho_a} \right) w_{ab}. \tag{9}$$

The last term including the parameter $\epsilon \approx 0.5$ is the so-called XSPH correction proposed by Monaghan [16], which ensures that neighboring particles move at approximately the same velocity. It prevents particles with different velocities from occupying nearly the same location.

2.3 *Equation of state*

In Eq. (5), pressure is unknown and should be determined in advance. To avoid solving the Poisson's equation of pressure, particles are assumed to be weakly compressible in this study. The equation of state is then considered to determine pressure. This equation is also modified to provide a slower speed of sound and is proper for simulating the bulk flow of fluid. The equation of state for determining pressure is denoted as:

$$P = B\left[\left(\frac{\rho}{\rho_o}\right)^{\gamma} - 1\right], \tag{10}$$

where $\gamma = 7$, $B = c_o^2 \rho_o/\gamma$ and c_o is the speed of sound at the reference density ($\rho_o = 1000$ kg/m^3). To use the equation of state in this study, the speed of sound should be about 10 times faster than the maximum fluid velocity to maintain fluid density change less than 1%. Also, the time increment should be small enough to satisfy the Courant-Friedrich-Levy condition.

2.4 *Subparticle scale turbulence model*

Artificial viscosity, originally used in the equation of motion has a few advantages. First, in free surface problems, it is a stabilizer in a numerical scheme. Second, artificial viscosity prevents the particle from interpenetrating [32]. It then preserves both linear and angular momentum and has an acceptable manner in the case of rigid body rotations [16]. By contrast, it has some limitations. It is a scalar viscosity that cannot consider the flow directionally [33], and this causes strong dissipation and affects shear stress in the fluid [32].

A realistic expression of viscosity is the laminar viscosity [34] and sub-particle scale (SPS) technique to model turbulence. Gotoh *et al.* [35] used the SPS approach to model turbulence in some types of particle methods, such as moving particle semi-implicit and incompressible SPH [36]. Recently, Dalrymple and Rogers [32] recently developed a compressible SPH approach to subparticle scaling of turbulence. The governing equation is the large eddy simulation formation for the equation of motion, developed through Favre-averaging ($\bar{f} = \bar{\rho f}/\bar{\rho}$)

$$\frac{\mathrm{D}\mathbf{u}}{\mathrm{D}t} = -\frac{1}{\bar{\rho}}\nabla\bar{P} + \mathbf{g} + \frac{1}{\bar{\rho}}(\nabla \cdot \rho\bar{\nu}_o\nabla)\mathbf{u} + \frac{1}{\bar{\rho}}\nabla \cdot \tau^*, \tag{11}$$

where τ^* is the SPS stress tensor with elements (in tensor notation), and

$$\frac{\tau^*_{i,j}}{\bar{\rho}} = 2v_t\tilde{S}_{ij} - \frac{2}{3}\tilde{S}_{kk}\delta_{ij} - \frac{2}{3}C_I\Delta^2\delta_{ij}. \tag{12}$$

The constant C_I is considered to be 0.0066 according to Blin *et al.* [37]. δ_{ij} is the Knonecker delta and $\tilde{S}_{kk}$ is SPS turbulence kinetic energy. The Favre-filtered rate of strain tensor is

$$\tilde{S}_{ij} = -\frac{1}{2}\left(\frac{\partial\tilde{v}_i}{\partial x_j} + \frac{\partial\tilde{v}_j}{\partial x_i}\right). \tag{13}$$

The turbulent viscosity is obtained by the Smagorinsky model [38] to determine the eddy viscosity :

$$v_t = (C_s \cdot \Delta)^2|\bar{S}|), \tag{14}$$

where the Smagorinsky constant, $C_s = 0.12$, Δ is the initial particle spacing and the local strain rate $|\bar{S}| = (2\bar{S}_{ij}\bar{S}_{ij})^{1/2}$. The SPS stress are discretized using the symmetric formulation given by Lo and Shao [39]:

$$\frac{1}{\rho}\nabla \cdot \tau^*|_i = \sum_j m_j \left(\frac{\tau_i^*}{\rho_i^2} + \frac{\tau_j^*}{\rho_j^2}\right) \cdot \nabla_i W_{ij}. \tag{15}$$

Dalrymple and Rogers [32] described the momentum equation (Eq. 12) in SPH notation using laminar viscosity and SPS turbulence model:

$$\frac{\mathrm{D}\mathbf{u}_a}{\mathrm{D}t} = -\sum_b m_b \left(\frac{P_b}{\rho_b^2} + \frac{P_a}{\rho_a^2}\right) \nabla_a w_{ab} + \mathbf{g} + \sum_b m_b \frac{4\nu_o \mathbf{u}_{ab}\mathbf{r}_{ab}}{|\mathbf{r}_{ab}^2|(\rho_a + \rho_b)} \nabla_a w_{ab} + \sum_b m_b \left(\frac{\tau_b}{\rho_b^2} + \frac{\tau_a}{\rho_a^2}\right) \nabla_a w_{ab} \tag{16}$$

where ν_o is the kinetic viscosity of laminar flow (10^{-6} m^2/s).

2.5 *Density reinitialization*

Oscillations exist in the predicted pressure field, so it is necessary to remove oscillations. A number of methods have been proposed (for an overview, see [40]). A computationally least expensive way is to employ a filter over the density of the particles and then re-assign a density to each particle (see [41]). There are two methods of correction, zeroth order and first order. The Moving Least Squares (MLS) approach was developed by [42] and successfully applied by [41] and [43]. More regular density distribution can be obtained using a MLS density filter (see [44]), so the MLS density filter is used in the proposed SPH model.

3 Numerical Model

In this condition, the water surface rises abruptly. Surface rollers are formed. Intense mixing occurs. Air is entrained. Hence, a large amount of energy is dissipated through frictional resistance of the channel. Hydraulic jumps in open-channel flow are characterized as a drop in the Froude number F_r

defined as

$$F_r = \frac{v}{\sqrt{gy}} \tag{17}$$

from supercritical ($F_r > 1$) to subcritical ($F_r < 1$) conditions. The result is a piecewise increase in depth y and a step decrease in flow velocity v passing through the jump. Fig. 1 shows a schematic of typical jump characteristics where E_1 is the energy of the upstream flow, E_2 is the energy of the downstream flow and L_j is the length of the hydraulic jump. It indicates that the channel flow with the depth of supercritical flow, y_1, jumps up to its subcritical conjugate depth, y_2. The result of this abrupt change is considerable energy loss and turbulence, E_L.

3.1 *Initial and boundary conditions*

Fig. 2 shows the physical flow domain in the proposed SPH model. It is based on the experiment conducted by [20]. A still water depth is 10 m. The opening of the gate is 1 m. The crested weir is 1 m high. The basin is 115 m long. Froude number in this case is approximately 3.5.

The gate, an obstacle and the bottom boundary are defined by lines of particles exerting repulsive forces on fluid particles (for the similar idea, see [45]). Central forces similar to those in molecular dynamics [46] are imposed in boundary particles. More details regarding the central force in boundary particles can be found in [23].

3.2 *Details of the basin*

Herein, a hydraulic jump on porous media was numerically studied for varying porosity and Froude number. A hydraulic jump is produced in a rectangular flume that is 1 m deep. The distance from the gate to the porous media is 13.88 m. The length of stilling basin is 125 m (see Fig. 3). Herein, a solid obstacle is replaced by porous media. Six-row staggered cylinders are used to model the porous media. In this study, the porous media have 1-m height and 4-m width. The diameter of cylinder was 0.16 m (see Fig. 4 for more details). Six-row staggered cylinders are used to model the porous media.

For an instance porosity(ϕ) = 0.85, transverse and longitudinal distance is $S_L \times S_T$= 4.0D $\times$ 2.1D, respectively. The equation of porosity is denoted as

$$\phi = 1 - \frac{V_s}{V_T} \tag{18}$$

where ϕ is porosity, V_s is the total volume of cylinders, and V_t is the total volume.

The discharge is 10.96 m^3/s. Water enters the flume under a sluice gate with a streamlined lip, producing a uniform super-critical stream with a thickness of y_1. A tailgate is used to control the tailwater depth in the flume. In the numerical simulations, the tailgate is adjusted so that the jumps are formed on the porous media (see Fig. 3).

The initial depth y_1 measured above the plane bed is equal to 1.76, 1.618, 1.452, 1.0, 0.846, and 0.788 m for different Froude numbers. Values of y_1 and V_1 are selected to achieve a wide range of the Froude numbers from 1.5 to 5.0. The Reynolds number $Re = V_1 y_1/\nu$ varies from $6.90E6$ - $1.095E7$. The considered porosities in this model are 0.0, 0.68, 0.78, 0.85, and 1.0.

4 Numerical Results and Discussion

4.1 *Validation for free hydraulic jump*

The schematic of the model for validation is illustrated in Fig. 2. The initial condition is designed to fit the experimental conditions of Lopez *et al.* [20]. The present computational system comprises fluid particles and walls. The model has initial height of water 10 m, opening of the gate 1 m, crested weir height 1 m, length of stilling basin 115 m. For this test, the Froude number of the flow upstream of the jump is approximately 3.5. It simulates with Monaghan's artificial viscosity $\alpha = 0.01$ and $\beta = 0.00$. Fluid particles are initially placed in a stagger grid with particle spacing dx= dz = 0.2 m. The total number of particles in the numerical model is 22,000 (including 2,723 boundaries particle). Lopez *et al.*[20] used 11,000 particles to simulate this problem. The simulation is performed in a workstation with two 3.40GHz Intel CPU and 3GB RAM. Computing the results up to the total time of 20 sec requires < 72 CPU hours.

Fig. 5 compares numerical with experimental [20] free surface profiles at difference time steps. The numerical prediction is presumably in good agreement with the experimental results.

4.2 *Validation hydraulic jump pass an obstacle*

The schematic of the model for validation using a solid obstacle is depicted in Fig. 3. The initial condition is created to fit the experimental conditions of Habibzadeh *et al.* [5]. The present computational system comprises fluid particles and walls. Initial height of water 10 m, opening of the gate of 1 m, and length of stilling basin 125 m are installed. A weir crest is not installed in this model. In addition, the distance from the gate 13.88 m, wall thickness 0.79 m, and height of obstacle 1.58 m. For this validation, the Froude number of the flow upstream of the jump varies between 4.0 and 7.0. The SPS turbulence model is used. Fluid particles are initially placed in a stagger grid with the particle spacing dx= dz = 0.05 m. The total number of particles in the numerical model is 158,546 (including 59,195 boundaries particles). In this case, the simulation is performed in a workstation with two 3.40GHz Intel CPU and 3GB RAM. Computing the result up to the total time of 20 sec requires < 120 CPU hours.

The energy dissipation efficiencies of the hydraulic jump with a solid obstacle are compared between experiments by Habibzadeh *et al.* [5] and the present SPH model (Fig. 6). The energy loss is increased by the increasing Froude number. This is because the obstacle enhances energy loss by increasing the mixing in the jump (see [5]).

In addition, Table 1 presents a quantitative comparison among the experimental data [5], SPS turbulence model, and artificial viscosity model. The results indicate that the average discrepance between the SPS turbulence model and experimental data is approximately 10% whereas it is 20% between the artificial viscosity model and experimental data.

4.3 *Characteristics of hydraulic jump*

The effects of porous media on the hydraulic jumps are under dispute. The most crucial relevant parameters include the conjugate depth ratio y_2/y_1, energy dissipation ratio of the jump $\Delta E/E_1$, and the bottom shear stress.

4.3.1 Conjugate depth ratio

The variations in the conjugate depth ratio y_2/y_1 with initial Froude number F_r and with different porosities (ϕ) are presented in Figs. 7(a) and (b). In general, for all cases, as F_r increases (moves toward more supercritical flow), the conjugate depth ratio also increases. The porosity of the obstacle affects the conjugate depth ratio as shown in Fig. 7(a). For the supercritical flow in a horizontal rectangular channel, the energy of flow is dissipated through frictional resistance of the channel and porous media. Therefore, velocity of flow decreases. The conjugate depth also decreases. The main reason for this effect is the increase of the shear stress not only in basin but also in the porous media. Another reason is that an obstacle increases the energy loss by improving the mixing in the jump (see [5]). The results are in a reasonable agreement with the experimental study proposed by [5].

Overall, the conjugate depth ratio of porosity 0.68 is much lower than that of others. For instance, for $F_r \leq 1.5$, the reduction in the conjugate depth ratio is of approximately 30.5%, whereas it is approximately 22.1% for $F_r \geq 5.0$.

4.3.2 The energy dissipation of the jump

One of the most critical issues related to the hydraulic jumps is energy dissipation in channels, dam spill ways and similar structures, such that the excess kinetic energy does not damage these structures. The rate of energy dissipation or head loss across a hydraulic jump is a function of F_r. Chanson [47] indicated that the head loss can be increased by increasing F_r.

The efficiency of the hydraulic jump η , can be expressed as 100% - $(E_1 - E_2)/E_1$, where $E_1 =$

$y_1 + Q^2/(2gy_1^2)$ and $E_2 = y_2 + Q^2/(2gy_2^2)$ are the energy before and after the jump, respectively. In Fig. 8, the relative energy loss E_L/E_1 where $E_L = E_1$ - E_2 is plotted versus different Froude numbers. Fig. 8 indicates that when the Froude number increases, the energy dissipation of the hydraulic jump in the proposed SPH model also increases because the energy dissipation is a function of Fr. In addition, Fig. 8 demonstrates that for similar Froude numbers for all models, the energy loss of a jump with an obstacle and porous media is higher than that without an obstacle or porous media. The findings of the current study are consistent with [5].

The difference between the energy dissipation for jumps on the porosity of 0.68 and the case without an obstacle is approximately 27.3% at $F_r < 5.0$. On the other hand, when the $F_r > 1.5$, the difference of energy dissipation of the model between the case without an obstacle and with the porosity of 0.68 is $<$ 24.3%.

4.3.3 The bottom shear stress

The resulting shear stress can be estimated using the SPS approach for turbulence modeling, first described by [35].

Fig. 9 presents the comparison of bottom shear stress in upstream for different models using the second-order polynomial. The shear stress profiles in upstream for various porosity are obtained by Eq. 13 and compared with the model without an obstacle. Fig. 9 depicts the second-order polynomial curve fitting of points, which is the best fitting in each case and shown in line types. Fig. 9 also indicates that the bottom shear stress upstream (before obstacle or porous media) of the hydraulic jump nearby the gate is higher than that near an obstacle. The shear stress due to a solid obstacle is larger than that due to others. This is due to the interaction between the supercritical stream with an obstacle.

The comparisons of the bottom shear stresses in the downstream of an obstacle for different areas are also presented in Fig. 10. The turbulence is produced by the vorticity, induced by solid matrix walls (see [11]). Higher shear stress means stronger turbulence. The highest bottom shear stress occurs when the porosity is 0.85. Therefore, at this porosity, the flow is more turbulent. The results in Area B is in contrary with Area A. The shear stress of the porosity of 0.68 is higher than the others. For instance, in $x = 81$ and 89 m the highest bottom shear stress is $3.9E-6$ and $1.62E-6$ $\mathrm{m^2/sec^2}$ ($\phi = 0.68$), respectively, and the lowest bottom shear stress is $0.9E-6$ and $0.6E-6$ $\mathrm{m^2/sec^2}$ (without obstacle), respectively. This is due to the wall resistance after the recirculation area.

Fig. 11 compares velocity profiles for different models (a) without an obstacle, (b) with a solid obstacle, or (c) with a porosity of 0.85. A higher mixing flow is noted in the porous media model (Fig. 11,c). The main reason of this difference is the higher turbulence effect in the porous medium model. As the velocity is maximized at the front, the large vorticity is induced between two cylinders in porous media. Furthermore, the vorticity becomes large between two flat plates and the narrow gap is formed. When the streaks with the disturbances pass through the throat, they are exposed to the large vortices. The wall effect produces swirl or mixing flow. This finding agrees with the experimental findings of Takatsu and Masuoka [11]. In addition, the discharge capacity of channels with a permeable region is highly reduced for turbulent flow and high permeability. The findings of the current study are consistent with [48] who mentioned the same result.

The effect of porosity on the shear stress distributions, depicted in Fig. 12, indicates that the shear stress for the porosity of 0.68 is larger than that for the porosities of 0.78 and 0.85 as shown in circulated regions. The main reason for this difference is the mixing flow due to turbulence.

5 Conclusions

The SPH model has been established to simulate characteristics of the hydraulic jump interacting porous media. Five models including cases without an obstacle, with a solid obstacle, or porous media have been considered to study the effect of porous media on the hydraulic jump. The free surface should rise after a hydraulic jump. The rising water level or the so-called conjugate depth ratio decreases when porous media are applied. Moreover, the lowest conjugate depth ratio occurs at the porosity of 0.68. In other words, porous media can shorten the hydraulic jump most effectively. The shear stress at the bottom is determined along the flow direction. Porous media induce more shear stress than do those without an obstacle. The maximum shear stress distribution occurs when the porosity is 0.68. This finding agrees with the results of an experimental study on hydraulic jumps in a solid obstacle by [5]. The energy dissipation in the hydraulic jump is calculated as well. Regarding the numerical results, porous media can dissipate more energy than those with or without an obstacle. The interaction of the supercritical stream with porous media produces a high shear stress level and therefore more turbulence. Among the five models, for the porosity of 0.68, the energy dissipation in the hydraulic jump is maximum. In conclusion, this study may provide useful information and an effective tool aiding engineers in designing a porous medium obstacle to prevent the damage resulting from the hydraulic jump in an open channel.

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Table 1: Validation for the submerged obstacle case.

F_1	E_L/E_1 (Experiment)	E_L/E_1 (SPS Turbulence)	Discrepancy presentage Experiment & SPS (%)	E_L/E_1 Artificial viscosity	Discrepancy presentage Experiment & Artificial viscosity (%)
4.0	42.66	38.87	9.75	31.21	36.69
4.5	48.01	42.65	12.54	37.20	29.06
5.0	53.49	46.97	13.88	42.43	26.07
6.0	60.55	54.53	11.04	50.28	20.43
7.0	65.98	62.00	6.42	56.64	16.49