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Design Optimal Linear Quadratic Regulator For Static Synchronous Compensator (Statcom) Using Particle Swarm Optimization

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Abstract — **Static Synchronous Compensator (STATCOM) can improve power factor and dynamic stability on power system. STATCOM is Multiple Input and Multiple Output (MIMO) system, which can be presented by a mathematical model. This paper presents an application of Particle Swarm Optimization (PSO) as a modern tool to design controller parameter of weighting matrices Q and R of the LQ optimal control to obtain optimal gain as feedback of the Static Synchronous Compensator (STATCOM) in its optimization. The LQ optimal control is designed using PSO to improve the dynamic response of the Static Synchronous Compensator (STATCOM). The final result show that application of LQ optimal control designed by using PSO can suppress active current I_d, reactive current I_q, and DC voltage V_{dc} in the shortest settling time among the application of the alternative of proposed control.**

Keywords— **STATCOM, linear quadratic (LQ) optimal control, particle swarm Optimization (PSO).**

I. INTRODUCTION

The application of Flexible AC Transmission System (FACTS) devices are intended to improve the quality of power system. One of type FACTS devices are Static Synchronous Compensator (STATCOM). The function of Static Synchronous Compensator (STATCOM) is to improve power factor, voltage, and also to enhance the stability of the system. STATCOM is Multiple Input and Multiple Output (MIMO) system, which needed design multivariable control [1].

From several of the research has done earlier confirms the advantages of STATCOM as one of FACTS device [2,3,4] which includes,

a. Weigth, size, and cost is inexpensive;
b. Able to operated in lagging and leading condition;
c. Controlling reactive power with fast response;
d. Able to used as an active harmonic filter.

In the past years, the analysis of dynamic and control has been done. The performance of dynamic STATCOM is depend on its dynamic model. The research of the performance of STATCOM in operating condition and steady-state response has done by researcher [4,5].

Some of researchers has designed the controller for steady-state performance of STATCOM based on vector state (phasor) diagram analysis to power system quantities [6,7]. One of control approach for this application is using a Proportional-Integral (PI) control method [8].

The LQ optimal control method is a control which have purpose to provide an optimal gain feedback with the minimum J index performance value [9,10].

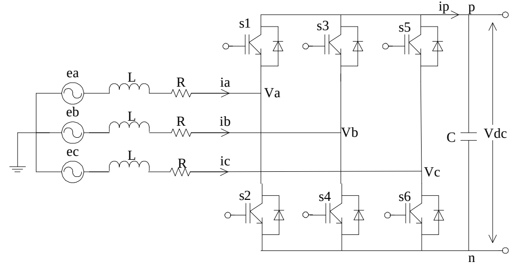
Particle Swarm Optimization (PSO) is a stochastic optimization technique based population, this method is classified as an adaptive method that used to solve complex optimization problem. PSO can provide an excellent improving for the system performance [11, 12].

In this paper, the LQ optimal control is used to obtain the optimal gain K as a feedback for Static Synchronous Compensator (STATCOM). The LQ optimal control is designed using Particle Swarm Optimization (PSO) to obtain optimal gain K through Algebraic Riccati Equation (ARE) solution. Particle Swarm Optimization (PSO) is a tool for tuning Q and R weighting matrices parameter in the LQ optimal control.

II. MODELLING OF STATCOM

Generally, the modeling of the Static Synchronous Compensator (STATCOM) which used in practice is the STATCOM based Voltage Source Converter (SVC). The modeling of the circuit Static Synchronous Compensator (STATCOM) describe in Figure 1 [13].

Fig.1, The equivalent circuit of *Static Synchronous Compensator* (STATCOM) [1,13]



where,

i_a, i_b, i_c = line current;
V_a, V_b, V_c = converter phase voltage;
e_a, e_b, e_c = AC source phase voltage;
$Vdc=Vpn$ = DC side voltage;
i_p = DC side current;
L = inductance of the line reactor;
R = resistance of the line reactor;
C = DC side capacitor,

Acorrding Ref [13] the linear mathematic model of the STATCOM as follow:

$$\frac{d}{dt}\begin{bmatrix} i_d \\ i_q \\ Vdc \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{D_d}{3L} \\ -\omega & -\frac{R}{L} & -\frac{D_q}{3L} \\ \frac{3}{2C}D_d & \frac{3}{2C}D_q & 0 \end{bmatrix}\begin{bmatrix} i_d \\ i_q \\ Vdc \end{bmatrix} + \begin{bmatrix} \frac{1}{3L} \\ 0 \\ 0 \end{bmatrix} Vm \quad (1)$$

For nonlinear system, the dynamic system explained as state space model which can be written as follow

$$\dot{x}(t) = \mathbf{A}\,x(t) + \mathbf{B}\,u(t) + \mathbf{L}\,d(t) \quad (2)$$

$$y(t) = \mathbf{C}x(t) \quad (3)$$

with,

$$\mathbf{A} = \frac{\partial f}{\partial x} \,|\, x^0, u^0$$

$$\mathbf{B} = \frac{\partial f}{\partial u} \,|\, x^0, u^0$$

If we set $x = x^0 + \delta x$ and $u = u^0 + \delta u$ as the states we can obtain the new linear system as follow,

$$\delta\dot{x} = \mathbf{A}\delta x + \mathbf{B}\delta u$$

Using Jacobian matrix method, we have

$$\mathbf{A} = \frac{\partial f}{\partial x} \,|\, (x^0, u^0) = \begin{bmatrix} \frac{-R}{L} & \omega & \frac{-D_{d0}}{3L} \\ -\omega & \frac{-R}{L} & \frac{-D_{q0}}{3L} \\ \frac{3D_{d0}}{2L} & \frac{3D_{q0}}{2L} & 0 \end{bmatrix} \quad (4)$$

$$\mathbf{B} = \frac{\partial f}{\partial u} \,|\, (x^0, u^0) = \begin{bmatrix} \frac{-V_{dc0}}{3L} & 0 \\ 0 & \frac{-V_{dc0}}{3L} \\ \frac{3i_{d0}}{2C} & \frac{3i_{q0}}{2C} \end{bmatrix} \quad (5)$$

where,

A = system matrix;
B = input matrix;
C = measurement matrix;
L = disturbance matrix;
x(t) = state variable, $\in \Re^n$;
u(t) = input variable, $\in \Re^m$;
y(t) = output variable, $\in \Re^r$,

By using the STATCOM mathematic model in equation (2), which are that system is open loop system. With a loop, we can build the closed full state feedback loop as shown in Figure 2.

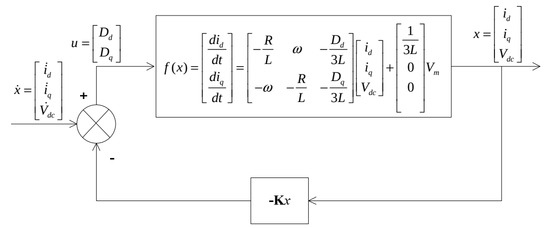


Fig.2, Closed loop control diagram of *Static Synchronous Compensator* (STATCOM)

III. OPTIMAL CONTROL

State space equation at eq. (2) can be solved to obtain an optimal gain through a linear quadratic optimal control solution using a quadratic cost *J* as follow [1],

$$J = \frac{1}{2}\int_{t_0}^{t_f} [x^T(t)\mathbf{Q}x(t) + u^T(t)\mathbf{R}(t)]dt \quad (6)$$

The optimal solution of eq. (31) using eq. (35) as a criteria is given as follow,

$$\dot{S} = \mathbf{A}^T S + S\mathbf{A} - S\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T S + \mathbf{Q} \quad (7)$$

If a time varying positive symmetric matrix *S(t)* converge at $t_f = \infty$, the solution of eq. (36) can be obtain as an *Algebraic Riccati Equation* (ARE) matrix as follow,

$$0 = \mathbf{A}^T S + S\mathbf{A} - S\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^T S + \mathbf{Q} \quad (8)$$

The gain **K** can be written as follows,

$$\mathbf{K} = \mathbf{R}^{-1}\mathbf{B}^T S \text{ and } u = -\mathbf{K}x \quad (9)$$

Then eq. (31) can be written as a closed loop system so that we can discuss the converging behavior using the following equation,

$$\dot{x} = (\mathbf{A} - \mathbf{BK})x \quad (10)$$

IV. PARTICLE SWARM OPTIMIZATION

PSO has several of the special characteristic computation as below,

1. PSO is initialized by a populating of random solutions;
2. PSO searching an optimum solution through generation updating;
3. The updating of PSO based on previous generation,

The mathematic equation based on the above PSO concept can be written as follow,

The velocity updating equation,

$$v_i^{k+1} = v_i^k + c1r1\left(pBest_i - x_i^k\right) + c2r2\left(gBest - x_i^k\right) \quad (11)$$

The position updating equation,

$$x_i^{k+1} = x_i^k + v_i^{k+1} \quad (12)$$

Particle Swarm Optimization (PSO) algorithm:

1. Initialize a group of particles with random position and their related velocities to fulfill the inequality constraints;

2. Check for the satisfaction of the equality constraint and adjust the solution if required;
3. Evaluate the fitness function of each particle;
4. Compare the current value of the fitness function with the particle previous best value (*pBest*). If the current fitness value is less, then allocate the current fitness value to *pBest* and allocate the current coordinates (position) to *pbest*;
5. Determine the current global minimum fitness value among the current positions;
6. Compare the current global minimum with the previous global minimum (*gBest*). If the current global minimum is better than *gBest*, then allocate the current global minimum to *gBest* and allocate the current coordinates (positions) to *gBest*;.
7. Change the velocities according to equation (11);
8. Move each particle to the new position according to equation (12) and return to step 2;
9. Repeat step 2-8 until a stopping criterion as satisfied or the maximum number of iterations is achieved.

V. PROPOSED CONTROL

In this study, the LQ optimal control is used to obtain the optimal gain **K** as a feedback of the *Static Synchronous Compensator* (STATCOM). The LQ optimal control is designed using *Particle Swarm Optimization* (PSO) to obtain optimal gain **K** through *Algebraic Riccati Equation* (ARE) solution. *Particle Swarm Optimization* (PSO) is a tool for tuning **Q** and **R** weighting matrices parameter in the LQ optimal control.

The structure of particles in PSO algorithm which used in this study is represents the weight matrix **Q** and **R**. The particle is divided into two parts: a) Part 1, a particle is includes several of dimension, which represent the weight matrix **Q** with the element size is 3x3 depends on amount of state variables of the system in size 3x3 elements (see eq. (29)), b) Part 2, a particle is consist several of dimension, which represent the weight matrix **R** with the element size is 1x1 depends on amount of inputs variables of the system in size 1x1 elements (see eq. (29)).

The equation (35) is the cost function of the system (*J*) which is used for objective function in optimization. The optimal gain **K** provided using the *Algebraic Riccati equation* (ARE) solution through PSO tuning is described in Figure 4.

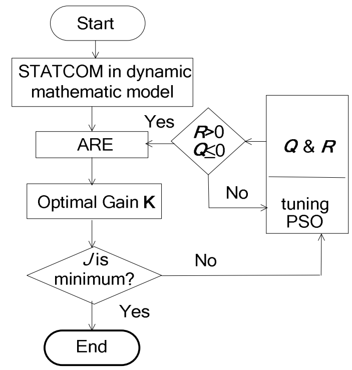


Fig.3, Flowchart of research

Vi. PERFORMANCE OF RESULTS

The optimal gain **K** used as a feedback control for *Static Synchronous Compensator* (STATCOM) is obtained using LQ optimal control. The parameters of LQ optimal control are the weighting matrices **Q** and **R** tuned by using *Trial-Error Method* (TEM) and *Particle Swarm Optimization* (PSO).

The result of tuning the weight matrix **Q** and **R** using *Particle Swarm Optimization* (PSO) as follow,

The weight matrix **Q**,

$$\mathbf{Q}_{PSO} = \begin{bmatrix} 0.1161 & 0 & 0 \\ 0 & 0.0558 & 0 \\ 0 & 0 & 0.0638 \end{bmatrix}$$

The weight matrix **R**,

$$\mathbf{R}_{PSO} = [0.0051]$$

The optimal gain **K** obtained using this method are,

$$\mathbf{K}_{PSO} = [5.5757 \quad \text{-}1.3983 \quad \text{-}2.5319]$$

The values of PSO parameter used are 0.9 for the inertia weight w, c1 and c2 selected as 2, 20 for the number of swarm, and 30 for the maximum number iteration.

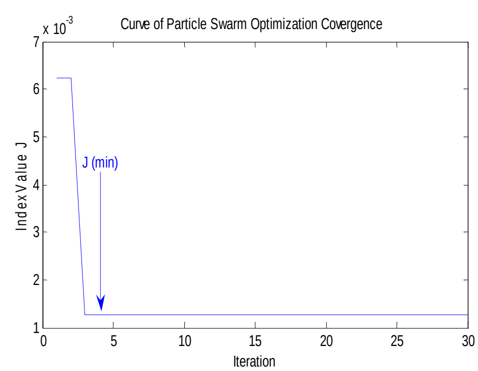


Fig.4, The convergence of PSO calculation

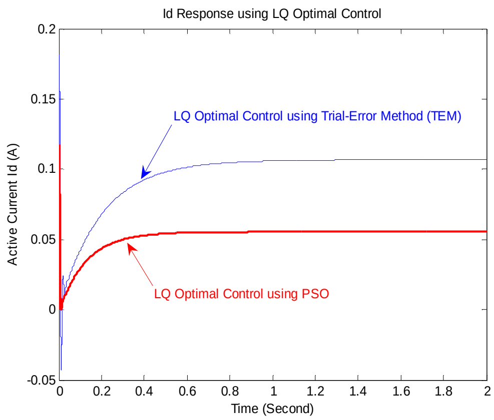


Fig.5, The dynamic response of the active current I_d.

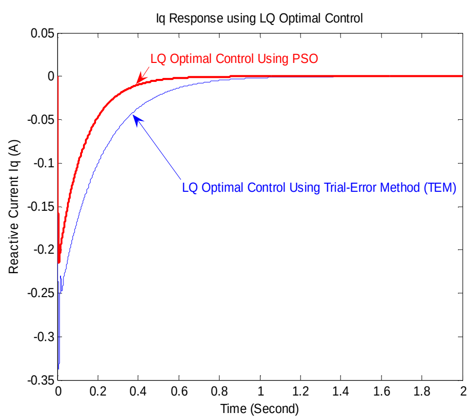


Fig.6, The dynamic response of reactive current I_q.

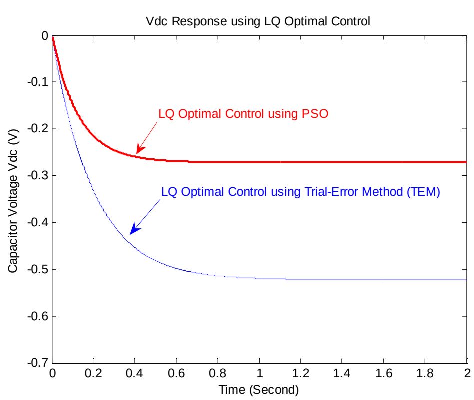


Fig.7, The dynamic response of DC voltage V_{dc}.

The convergence of PSO has reached at 3rd iteration and *J* index minimum can be reached at 3rd iteration described in Fig.8. The best value of *J* cost function resulted using PSO calculation is smaller (minimum) than using TEM calculation. The cost function of *J* minimum index performance using TEM calculation and PSO calculation are J_{TEM}=0.0076, and J_{PSO}=0.001267.

From simulation and analysis has been done, the closed loop dynamic response of the STATCOM are described in Fig.9, Fig.10, and Fig.11 we obtain,

a. The eigenvalues of the system,
   TEM calculation :
   1.0e+002 *
   -1.5416 + 3.7656i;
   -1.5416 - 3.7656i;
   -0.0500;
   PSO calculation :
   1.0e+002 *
   -3.3514 + 3.0477i;
   -3.3514 - 3.0477i;
   -0.0778;
b. The active current I_d *settling time* dynamic response using TEM and PSO are 1.198 s, and 0.6164 s;
c. The reactive current I_q *settling time* dynamic response using TEM and PSO are 1.115 s, and 0.8602 s;
d. The DC voltage V_{dc} *settling time* dynamic response using TEM and PSO are 1.233 s, and 0.7065 s;
e The *overshoot* of active current I_d and reactive current I_q are very small.

VII. CONCLUSION

In this paper, an application of *Particle Swarm Optimization* (PSO) to design the weight matrix **Q** and **R** to obtain the optimal gain **K** using LQ optimal control to design feedback of the STATCOM has been done. In the *overshoot* and *settling time* improvement, the application of PSO computation to design LQ optimal control to obtain optimal gain controller has shown extremely better result than alternative proposed method application such as no control or open loop and *Trial-Error Method* (TEM).

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